Principle-based use of digital technology to improve STEM learning

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There's much learning still to be done

Forces on a steel ball falling and bouncing off a steel table?

- falling
- stationary
- rising
There’s much learning still to be done

Forces on a steel ball falling and bouncing off a steel table?

- Falling
- Stationary
- Rising
There’s much learning still to be done

Forces on a steel ball falling and bouncing off a steel table?

- weight falling
- stationary
- rising
There’s much learning still to be done

Forces on a steel ball falling and bouncing off a steel table?

- Falling
- Stationary
- Rising
There’s much learning still to be done

How big is $N$—according to students?

- Falling
- Weight
- $N = ?$
- Stationary
- Weight
- Rising
- Weight
There’s much learning still to be done

Students mostly think $N = \text{weight}$:

- **Falling**: $N = \text{weight}$
- **Stationary**: $N = \text{weight}$
- **Rising**: $N = \text{weight}$
There’s much learning still to be done

(rest rock on hand)

$F_{onto\ hand} = weight$
There’s much learning still to be done

(rest rock on hand)

\[ F_{\text{onto hand}} = \text{weight} \]

(drop rock on hand)

\[ F_{\text{onto hand}} = \text{weight?} \]
and improvement of teaching, not least in online courses

Instructional quality of 50 MOOCs (Coursera, Udacity, EdX):

0 8 (median) (best) 72
3 25

“[T]he majority of MOOCs scored poorly on most instructional design principles. However, most MOOCs scored highly on organisation and presentation of course material. The results indicate that although most MOOCs are well-packaged, their instructional design quality is low.”

(Margaryan, Bianco, and Littlejohn, 2015)
Digital tools are a means, learning is the end, and principles can help us get there

1. **An organizing framework**

2. **Illustrations**
   
   a. Constructive visualizations
   
   b. Constructing formula explanations
   
   c. Adaptive hints
   
   d. Collaborative reading
The ICAP framework can help us improve learning

**ICAP hierarchy:**

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interactive</strong></td>
<td>dialogue with tutor or another student</td>
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<tr>
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</tr>
<tr>
<td><strong>Passive</strong></td>
<td>paying attention (watching lecture or video)</td>
<td>minimal understanding</td>
</tr>
</tbody>
</table>

(Chi and Wylie, 2014)
The ICAP framework can help us improve learning

1. ICAP framework
2. Illustrations
   a. Constructive visualizations
   b. Constructing formula explanations
   c. Adaptive hints
   d. Collaborative reading
Standard problems need recasting to benefit from digital’s affordances

Section 6.1

Find $\theta$

Week 6 introduction

Make a regular tetrahedron
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
&C_x = 1 \\
&C_y = 1 \\
&D_x = 1 \\
&D_y = 1 \\
&D_z = 1
\end{align*}
\]

\[
\begin{align*}
&AB = 1 \\
&AC = 1.4142 \\
&BC = 1.0000 \\
&AD = 1.7321 \\
&BD = 1.4142 \\
&CD = 1.0000
\end{align*}
\]
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.7575 \\
C_y &= 1 \\
D_x &= 1 \\
D_y &= 1 \\
D_z &= 1
\end{align*}
\]

AB = 1
AC = 1.2545
BC = 1.0290
AD = 1.7321
BD = 1.4142
CD = 1.0290
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[ C_x = 0.5, \quad C_y = 1, \quad D_x = 1, \quad D_y = 1, \quad D_z = 1 \]

- \( AB = 1 \)
- \( AC = 1.1180 \)
- \( BC = 1.1180 \)
- \( AD = 1.7321 \)
- \( BD = 1.4142 \)
- \( CD = 1.1180 \)
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.5 \\
C_y &= 0.9325 \\
D_x &= 1 \\
D_y &= 1 \\
D_z &= 1 \\
AB &= 1 \\
AC &= 1.0581 \\
BC &= 1.0581 \\
AD &= 1.7321 \\
BD &= 1.4142 \\
CD &= 1.1201
\end{align*}
\]
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.5 \\
C_y &= 0.8675 \\
D_x &= 1 \\
D_y &= 1 \\
D_z &= 1
\end{align*}
\]

- \(AB = 1\)
- \(AC = 1.0013\)
- \(BC = 1.0013\)
- \(AD = 1.7321\)
- \(BD = 1.4142\)
- \(CD = 1.1259\)
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

- $C_x = 0.5$
- $C_y = 0.8675$
- $D_x = 0.7575$
- $D_y = 1$
- $D_z = 1$

AB = 1
AC = 1.0013
BC = 1.0013
AD = 1.6043
BD = 1.4349
CD = 1.0411
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\begin{align*}
C_x &= 0.5 \\
C_y &= 0.8675 \\
D_x &= 0.5 \\
D_y &= 1 \\
D_z &= 1
\end{align*}

AB = 1  \\
AC = 1.0013  \\
BC = 1.0013  \\
AD = 1.5000  \\
BD = 1.5000  \\
CD = 1.0087
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.5 \\
C_y &= 0.8675 \\
D_x &= 0.5 \\
D_y &= 0.655 \\
D_z &= 1 \\
AB &= 1 \\
AC &= 1.0013 \\
BC &= 1.0013 \\
AD &= 1.2958 \\
BD &= 1.2958 \\
CD &= 1.0223
\end{align*}
\]
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.5 \\
C_y &= 0.8675 \\
D_x &= 0.5 \\
D_y &= 0.2875 \\
D_z &= 1
\end{align*}
\]

\[
\begin{align*}
AB &= 1 \\
AC &= 1.0013 \\
BC &= 1.0013 \\
AD &= 1.1544 \\
BD &= 1.1544 \\
CD &= 1.1560
\end{align*}
\]
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[ C_x = 0.5 \]
\[ C_y = 0.8675 \]
\[ D_x = 0.5 \]
\[ D_y = 0.2875 \]
\[ D_z = 0.925 \]

\[ AB = 1 \]
\[ AC = 1.0013 \]
\[ BC = 1.0013 \]
\[ AD = 1.0901 \]
\[ BD = 1.0901 \]
\[ CD = 1.0918 \]
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[ \begin{align*} C_x &= 0.5 \\ C_y &= 0.8675 \\ D_x &= 0.5 \\ D_y &= 0.2875 \\ D_z &= 0.8175 \end{align*} \]

- AB = 1
- AC = 1.0013
- BC = 1.0013
- AD = 1.0005
- BD = 1.0005
- CD = 1.0024
Constructive visualizations promote minds-on learning

To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[
\begin{align*}
C_x &= 0.5 \\
C_y &= 0.8675 \\
D_x &= 0.5 \\
D_y &= 0.2875 \\
D_z &= 0.8175 \\
AB &= 1 \\
AC &= 1.0013 \\
BC &= 1.0013 \\
AD &= 1.0005 \\
BD &= 1.0005 \\
CD &= 1.0024
\end{align*}
\]
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To get a feel for where the hydrogens lie, make a regular tetrahedron using the visualization below. Move the vertices (the hydrogens) until you have a reasonably precise regular tetrahedron.

\[ C_x = 0.5 \]
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\[ D_z = 0.8175 \]

AB = 1
AC = 1.0013
BC = 1.0013
AD = 1.0005
BD = 1.0005
CD = 1.0024
Constructive visualizations belong to a learning hierarchy

**ICAP hierarchy:**

*Interactive*  dialogue with tutor or another student  →  understanding that might create new ideas

*Constructive*  adding to the information (drawing a concept map or diagram, self-explaining)  →  deeper understanding that might transfer

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(Chi and Wylie, 2014)
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   a. Constructive visualizations
   b. **Constructing formula explanations**
   c. Adaptive hints
   d. Collaborative reading
Constructing formula explanations promotes minds-on learning
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES  (1 point possible)

Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \frac{\theta^2}{2}$. Enter your explanation by dragging and dropping to fill in the equation below.

$$\cos \theta = \left[ 1 - (\square) \right]^2 \approx 1 \square \square \square \square \text{ or } -?$$
Constructing formula explanations promotes minds-on learning

**COSINE AT SMALL ANGLES** (1 point possible)

Use the small-angle approximation \( \sin \theta \approx \theta \) to justify the related small-angle approximation \( \cos \theta \approx 1 - \theta^2 / 2 \). Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - \left( \theta^2 \right)^2 \right] \approx 1 - \theta^2 / 2 + \text{or} -? \]

\[
\sin \theta \quad \theta \quad \theta^2 \quad \frac{\theta^2}{2} \quad + \quad -
\]
Constructing formula explanations promotes minds-on learning

**Cosine at Small Angles** (1 point possible)

Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \theta^2 / 2$. Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - (\text{□})^2 \right] \approx 1 - \frac{\theta^2}{2}.
\]
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES (1 point possible)

Use the small-angle approximation \( \sin \theta \approx \theta \) to justify the related small-angle approximation \( \cos \theta \approx 1 - \frac{\theta^2}{2} \). Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - \left( \sin \theta \right)^2 \right] \approx 1 - \frac{\theta^2}{2} + \text{or} -?
\]

\[
\sin \theta \quad \theta \quad \theta^2 \quad \frac{\theta^2}{2} \quad + \quad -
\]

You have used 0 of 10 submissions
Constructing formula explanations promotes minds-on learning.

COSINE AT SMALL ANGLES (1 point possible)

Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \theta^2 / 2$. Enter your explanation by dragging and dropping to fill in the equation below.

$$\cos \theta = \left[ 1 - (\sin \theta)^2 \right] \approx 1 - \frac{\theta^2}{2} \text{ or } -?$$

You have used 0 of 10 submissions
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Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \frac{\theta^2}{2}$. Enter your explanation by dragging and dropping to fill in the equation below.

$$\cos \theta = \left[ 1 - (\sin \theta)^2 \right] \approx 1 - \frac{\theta^2}{2} + \text{ or } -?$$
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES  (1 point possible)

Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \frac{\theta^2}{2}$. Enter your explanation by dragging and dropping to fill in the equation below.

$$\cos \theta = \left[ 1 - \left( \frac{\sin \theta}{\sin \theta} \right)^2 \right] \approx 1 - \frac{\theta^2}{2} \pm \frac{1}{2}$$
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES (1 point possible)

Use the small-angle approximation \( \sin \theta \approx \theta \) to justify the related small-angle approximation \( \cos \theta \approx 1 - \frac{\theta^2}{2} \). Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - (\underbrace{\sin \theta }_{\frac{\theta^2}{2}})^2 \right] \approx \underbrace{1}_{+ \text{ or } -?} - \frac{\theta^2}{2}.
\]

You have used 0 of 10 submissions
Constructing formula explanations promotes minds-on learning

**COSINE AT SMALL ANGLES**  
(1 point possible)

Use the small-angle approximation \( \sin \theta \approx \theta \) to justify the related small-angle approximation \( \cos \theta \approx 1 - \theta^2/2 \). Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - (\sin \theta)^2 \right] \approx 1 - \frac{\theta^2}{2} \]

You have used 1 of 10 submissions
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES  (1 point possible)

Use the small-angle approximation \( \sin \theta \approx \theta \) to justify the related small-angle approximation \( \cos \theta \approx 1 - \frac{\theta^2}{2} \). Enter your explanation by dragging and dropping to fill in the equation below.

\[
\cos \theta = \left[ 1 - \left( \frac{\sin \theta}{\theta} \right)^2 \right] \approx 1 - \frac{\theta^2}{2} \text{ or } -?
\]

You have used 1 of 10 submissions
Constructing formula explanations promotes minds-on learning

COSINE AT SMALL ANGLES  (1/1 point)

Use the small-angle approximation $\sin \theta \approx \theta$ to justify the related small-angle approximation $\cos \theta \approx 1 - \theta^2/2$. Enter your explanation by dragging and dropping to fill in the equation below.

$$\cos \theta = \left[ 1 - (\sin \theta)^2 \right] \approx 1 - \frac{\theta^2}{2} \quad \text{or} \quad -\frac{\theta^2}{2}.$$
Constructive activities belong to a learning hierarchy

ICAP hierarchy:

**Interactive**  
dialogue with tutor or another student  
→ understanding that might create new ideas

**Constructive**  
adding to the information (drawing a concept map or diagram, self-explaining)  
→ deeper understanding that might transfer

**Active**  
doing something with hands or bodies with the material, but not adding to the information  
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paying attention (watching lecture or video)  
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(Chi and Wylie, 2014)
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Adaptive hints lie between the constructive and interactive levels
Adaptive hints lie between the constructive and interactive levels

TAKING OUT THE BIG PART FROM A LARGE LOGARITHM (1 point possible)

For the following speed-calculation estimate, use the approximation, worth memorizing, that \( \ln 10 \approx 2.3 \) (accurate to 0.1%!). Are you ready?!

Take out the big part to estimate \( \ln 11 \) to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

-1

Hint: Logarithms of numbers less than 1 are negative.

You have used 2 of 15 submissions
Adaptive hints lie between the constructive and interactive levels

TAKING OUT THE BIG PART FROM A LARGE LOGARITHM  (1 point possible)

For the following speed-calculation estimate, use the approximation, worth memorizing, that
\[ \ln 10 \approx 2.3 \] (accurate to 0.1%!). Are you ready?!

Take out the big part to estimate \( \ln 11 \) to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

2.3

You have used 0 of 15 submissions
Adaptive hints lie between the constructive and interactive levels

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For the following speed-calculation estimate, use the approximation, worth memorizing, that $\ln 10 \approx 2.3$ (accurate to 0.1%!). Are you ready?!

Take out the big part to estimate $\ln 11$ to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

2.3

Hint: You are halfway there: You have found the big part! Just include the next correction.

You have used 1 of 15 submissions
Adaptive hints lie between the constructive and interactive levels.
Adaptive hints lie between the constructive and interactive levels.

TAKING OUT THE BIG PART FROM A LARGE LOGARITHM (1 point possible)

For the following speed-calculation estimate, use the approximation, worth memorizing, that \( \ln 10 \approx 2.3 \) (accurate to 0.1%!\). Are you ready?!

Take out the big part to estimate \( \ln 11 \) to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

2.397895

Tsk, tsk: You used a calculator!

You have used 5 of 15 submissions
Adaptive hints lie between the constructive and interactive levels

TAKING OUT THE BIG PART FROM A LARGE LOGARITHM (1 point possible)

For the following speed-calculation estimate, use the approximation, worth memorizing, that \( \ln 10 \approx 2.3 \) (accurate to 0.1%!). Are you ready?!

Take out the big part to estimate \( \ln 11 \) to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

2.5

That's not right. But a street fighter never gives up!

You have used 7 of 15 submissions
Adaptive hints lie between the constructive and interactive levels

For the following speed-calculation estimate, use the approximation, worth memorizing, that \( \ln 10 \approx 2.3 \) (accurate to 0.1%!). Are you ready?!

Take out the big part to estimate \( \ln 11 \) to within 0.1%. For extra kudos, make the estimate in less than 10 seconds and, for double kudos, in your head!

2.4

Yes! And the fractional error is only 0.088 percent!
Adaptive hints lie between the constructive and interactive levels

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Collaborative reading can reach the interactive level

NotaBene (nb.mit.edu)
(Zyto, Karger, Ackerman, and Mahajan, 2014)
Collaborative reading can reach the interactive level

1.1 Example 1: CDROM design

The first example is from electrical engineering and information theory.

How far apart are the pits on a compact disc (CD) or CDROM?

Divide finding the spacing into two subproblems: (1) estimating the CD’s area and (2) estimating its data capacity. The area is roughly \( (10 \text{ cm})^2 \) because each side is roughly 10 cm long. The actual length, according to a nearby ruler, is 12 cm; so 10 cm is an underestimate. However, (1) the hole in the center reduces the disc’s effective area; and (2) the disc is circular rather than square. So \( (10 \text{ cm})^2 \) is a reasonable and simple estimate of the disc’s pitted area.

The data capacity, according to a nearby box of CDROM’s, is 700 megabytes (MB). Each byte is 8 bits, so here is the capacity in bits:

\[
700 \times 10^6 \text{ bytes} \times 8 \text{ bits/byte} = 5.6 \times 10^9 \text{ bits.}
\]

Each bit is stored in one pit, so their spacing is a result of arranging them into a lattice that covers the \( (10 \text{ cm})^2 \) area. \( 10^9 \) pits would need \( 10^5 \) rows and \( 10^5 \) columns, so the spacing between pits is roughly

\[
d \approx \frac{10 \text{ cm}}{10^5} \approx 1 \mu\text{m.}
\]
Collaborative reading can reach the interactive level

That calculation was simplified by rounding up the number of bits from $5 \times 10^9$ to $10^9$. The factor of 2 increase means that 1 µm underestimates the spacing by a factor of $\sqrt{2}$, which is roughly 1.4. The estimated spacing is 1.4 µm.

Finding the capacity on a box of CDROM’s was a stroke of luck. But fortune favors the prepared mind. To prepare the mind, here is a divide-and-conquer estimate for the capacity of a CDROM, or of an audio CD, because data and audio discs differ only in how we interpret the information. An audio CD’s capacity can be estimated from three quantities: the playing time, the sampling rate, and the sample size (number of bits per sample).

Estimate the playing time, sampling rate, and sample size.

Here are estimates for the three quantities:

1. **Playing time.** A typical CD holds about 20 popular-music songs each lasting 3 minutes, so it play for about 1 hour. Confirming this estimate is the following piece of history. Legend, or urban legend, says that the designers of the CD ensured that it could record Beethoven’s Ninth Symphony. At most tempos, the symphony lasts 70 minutes.

2. **Sampling rate.** I remember the rate: 44 kHz. This number can be made plausible using information theory and acoustics.

   First, acoustics. Our ears can hear frequencies up to 20 kHz (slightly higher in youth, slightly lower in old age). To reproduce audible sounds with high fidelity, the audio CD is designed to store frequencies up to 20 kHz: Why ensure that Beethoven’s Ninth Symphony can be recorded if, by skipping on the high frequencies, it sounds like was played through a telephone line?

   Second, information theory. Its fundamental theorem, the Nyquist–Shannon sampling theorem, says that reconstructing a 20 kHz signal requires sampling at 40 kHz – or higher. High rates simplify the anti-alias filter, an essential part of the CD recording system. However, even an 80 kHz sampling rate exceeded the speed of inexpensive electronics when the CD was designed. As a compromise, the sampling-rate margin was set at 4 kHz, giving a sampling rate of 44 kHz.

3. **Sample size.** Each sample requires 32 bits: two channels (stereo) each needing 16 bits per sample. Sixteen bits per sample is a compromise between the utopia of exact volume encoding (infinity bits per sample

I like the bit about adjusting the estimate by a factor of root 2. I do calculations like this often but don’t adjust the results by the factors that I rounded with. This is a useful point and it could be worth emphasizing.

Doesn’t the CD’s shape come into play? It seems to be difficult to space them all the same considering the disc is circular. Or maybe not? I’m not sure. The estimate is still valid under the approximation that all pits are about the same distance away, I am just curious.

Also, the CD spins. Wouldn’t that mean that if the pits were spaced the same, the ones farther out would have to be processed much faster than the ones near the center.

Is this why vinyl records have a higher sampling rate? It is an analog medium as opposed to a digital one, but if I recall correctly, my records are ripped via turntable to 24-bit tracks. Is this due to simply having more pits?

I am curious how the factor of 2 underestimates by $\sqrt{2}$? Is it due to approximating the shape as a square?

We are assuming the pits are arranged on the CD in a grid. To make the calculations easier, we assumed there were 10^10 total bits. However, this is off by about a factor of 2. Carrying this factor of 2 through the calculations, we get that there would actually be $\sqrt{10^10/2}=10^5/\sqrt{2}$ rows, so $d = 10 \text{ cm}/(10^5/\sqrt{2}) = 10 \text{ cm} / 10^5 * \sqrt{2}$ um * $\sqrt{2}$ = $\sqrt{2}$ um. So because we underestimated the number of bits by a factor of 2, we overestimated the number of rows by a factor of $\sqrt{2}$ (and also the # of col by $\sqrt{2}$) and thus underestimated the distance by a factor of $\sqrt{2}$.

Yes, I think you’re on the right track. Evidence for approximating the shape as a square is at the bottom of page 4, where the text mentions that 10^10 pits would require 10^5 rows and 10^5 columns.

You are right, and it shows me that I should explicitly describe (and diagram) my approximate mental picture of a square lattice of pits.

I don’t understand why we would do this simplification which only makes us do more work later. Why not plug in 5*10^9 bits in the first place?
That calculation was simplified by rounding up the number of bits from $5 \times 10^8$ to $10^9$. The factor of 2 increase means that 1 µm underestimates the spacing by a factor of $\sqrt{2}$, which is roughly 1.4. The estimated spacing is 1.4 µm.

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   First, acoustics. Our ears can hear frequencies up to 20 kHz (slightly higher in youth, slightly lower in old age). To reproduce audible sounds with high fidelity, the audio CD is designed to store frequencies up to 20 kHz: Why ensure that Beethoven’s Ninth Symphony can be recorded if, by skimping on the high frequencies, it sounds like was played through a telephone line?

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Is the factor of 2 important in doing an approximation? The difference in the final result is 4 micrometers, and even then that’s also just an approximation.

I agree; how do we know what are appropriate approximations to make, and how do we know when the extra factors count?

This intuitively sounds right... it’s about twice the wavelength of red light, and I’m pretty sure I’ve seen red lasers on various disk-reader modifications.

This all makes sense.

I feel like this is common knowledge now. I’ve always thought CDs were 700 MB, DVDs 4 GB, etc.

how are there cd’s with different amounts of capacity? does that mean that the lattice is smaller?

what are the extremes of the size of storage and how is the price change relative. how do you make a cheap one vs a expensive one

this answers the previous question.

This sounds like a pretty specific way to estimate the CD’s capacity. This is probably just a new way of thinking that I’m not used to, but I would consider this a calculation more than an estimation.

Some things are easily estimated, like the area of the CD, but I do agree that things like the sample rate aren’t easily estimated, especially if you don’t have too much knowledge on what the sampling rate is

I see it as several estimations used to calculate an estimate value...if each value that you are using in your formula is estimated, you're not really calculating the actual value.
That calculation was simplified by rounding up the number of bits from $5 \cdot 10^9$ to $10^9$. The factor of 2 increase means that 1 µm underestimates the spacing by a factor of $\sqrt{2}$, which is roughly 1.4. The estimated spacing is 1.4 µm.

Finding the capacity on a box of CDROM’s was a stroke of luck. But fortune favors the prepared mind. To prepare the mind, here is a divide-and-conquer estimate for the capacity of a CDROM – or of an audio CD, because data and audio discs differ only in how we interpret the information. An audio CD’s capacity can be estimated from three quantities: the playing time, the sampling rate, and the sample size (number of bits per sample).

Estimate the playing time, sampling rate, and sample size.

Here are estimates for the three quantities:

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That’s if you burn a CD, right? Does anyone know why many more (legally downloaded) mp3’s can fit on a CD? Is that just data compression?

Yes, mp3 files are compressed audio files. Traditional CD formats are very similar to the way that vinyl records worked, it’s a ‘note-for-note’ style [vast over-simplification there], where as mp3 files are smaller to store, but require more computing power to read.

Most industry produced CD’s - like 2 disks “1 hits” - sit around this number per CD. Data compression shouldn’t change it much.

I was also curious about this and went to wikipedia to read a little about CDs. In it it’s stated that “Standard CDs have a diameter of 120 mm and can hold up to 80 minutes of uncompressed audio.” So my assumption is that there is no data compression for CDs?

I thought CDs hold more like 80 minutes? 60 seems like a steep under-estimate.

I know right! I’ve been burning music to CD’s for my car for years now. It’s definitely 80 minutes (120 MB), but this is an estimation I guess. And 1 hour is a pretty number.

Not a big deal, but I think most CD’s are 80 minutes?

That’s a cool fact that I didn’t know.

Interesting piece of info. Has the capacity remained constant?

Based on whether or not the capacity is 700mb vs. 650mb... the time varies between 74 and 80 minutes.

Are there actually several different accepted tempos to play a Beethoven symphony at?

Here is a challenge: We are assuming that we have never seen the box of a CD so we have never seen how many Megs are in a CD. Now, let’s assume we have never listened to an Audio CD. Nowadays, we generally load CD’s with Mp3s, which have compression, variable bit rate, and configurable frequency ranges... For what type of reader is this book intended? The derivation of unit conversions make it seem that this book is meant for high schoolers, but the assumption that the reader is familiar with audio formats makes it seem that the book is meant for readers with much more experience than average college students. The comments by other students in the class show that even MIT students can be unaware of digital audio formats...

Comments on page 3
That calculation was simplified by rounding up the number of bits from \( 5 \times 10^9 \) to \( 10^9 \). The factor of 2 increase means that 1 \( \mu \)m underestimates the spacing by a factor of \( \sqrt{2} \), which is roughly 1.4. The estimated spacing is 1.4 \( \mu \)m.

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3. **Sample size.** Each sample requires 32 bits: two channels (stereo) each needing 16 bits per sample. Sixteen bits per sample is a compromise between the utopia of exact volume encoding (infinity bits per sample). I understand that with approximation, you use what you have to get to a reasonable estimate efficiently. Some numbers you don’t have a feel for. Is it really possible to estimate anything?

   I’m glad this is anonymous. Can someone explain what sampling rate means?

   Sampling rate is the rate at which you sample the sound. To be more specific, a cd-rom only has a limited, finite amount of space for the pits. Therefore, we don’t have the luxury of capturing a sound wave purely as the sinusoids that you see in calculus. Instead, we have to resort to picking points along the sinusoidal sound wave at regular intervals until we can make out the shape of the function (what its amplitude is, what its frequency is, etc.). Obviously, the more points you record, the more it will look like the original sound wave. The tradeoff occurs because too high a sampling rate will record an unnecessary amount of points along the sound wave and waste precious memory space on the cd-rom.

   This really helped. Thanks.

   The explanation from a fellow student should help. And thanks for the comment. I should indeed explain, at least in passing, what sample rate means when I first use the term.

   how am i supposed to know this?

   Where do we get these numbers (20 kHz for hearing and the Nyquist Shannon theorem)? How do we proceed with the approximation when we don’t know numbers like these.

   My sense was that the first approximation, using just a ruler and what is on the CD box, is the one to use if you don’t have any expertise. But it might be tempting, if you do have it (you know some information theory), not to try to approximate at all but to instead start trying to find the exact answer. Part of what I took away from the second approximation was: even if you have the expertise, you can use it to approximate, rather than just diving in trying to find the exact answer.

   I also found the first approximation to be more intuitive. I had no idea what a sampling rate or sample size was so the second approximation was harder for me to comprehend. Is the second one supposed to be ideal for approximating or are we able to use whatever tactic works best for us personally?
That calculation was simplified by rounding up the number of bits from \(5 \times 10^6\) to \(10^7\). The factor of 2 increase means that 1 \(\mu\)m underestimates the spacing by a factor of \(\sqrt{2}\), which is roughly 1.4. The estimated spacing is 1.4 \(\mu\)m.

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   **I remember the rate:**
   
   Where can we learn more about this theorem?

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   **What does the 44kHz sampling rate refer to?**
   
   What does the 44kHz sampling rate refer to?
   
   **Explain theory?**
   
   I think when sampling a signal you need to sample it at least twice every period, ideally once at the top of the wave and once at the bottom. Otherwise it can look like a way slower frequency because you are losing so much data by sampling infrequently.

   Otherwise, you get aliasing and data loss.

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   **What can we learn more about this theorem?**

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   **Does the ability to estimate depend on this type of knowledge?**

   I would not expect most readers (not even engineers) to be very familiar with wave file formats. Are you using these figures only to illustrate unit conversions or are you telling the reader that he/she should know all of this common sense? Many readers might feel intimidated by this section, if it isn’t stated that the section is just meant to illustrate unit conversions.

   I would have been unable to come up with any of these estimates myself due to lack of knowledge on this field. Is there any other way I could have come up with these estimates?

   I agree, this is a little beyond common knowledge.

   I don’t quite understand how sampling rate directly can be correlated to the frequency at which we can hear sound. I am assuming that the disc reader gathers some information from the CD stores it and then plays it. I don’t think it plays directly as fast as it reads. Thus the acoustics analogy seems to not quite work. Pits can only relay a yes or no respone (1 or 0). How can it dictate volume, and key which cover a large range? Maybe I don’t know enough about acoustics and how sound and cd players work.

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Collaborative reading can reach the interactive level

ICAP hierarchy:

**Interactive**
- dialogue with tutor or another student
  → understanding that might create new ideas

**Constructive**
- adding to the information (drawing a concept map or diagram, self-explaining)
  → deeper understanding that might transfer

**Active**
- doing something with hands or bodies with the material, but not adding to the information
  → shallow understanding

**Passive**
- paying attention (watching lecture or video)
  → minimal understanding

(Chi and Wylie, 2014)
Digital tools are a means, learning is the end, and principles can help us get there

1. ICAP framework

2. Illustrations
   a. Constructive visualizations
   b. Constructing formula explanations
   c. Adaptive hints
   d. Collaborative reading
Further reading


Merrill, M. David (2002). *First principles of instruction.* *Educational Technology Research and Development* 50(3):43–59. *Another outstanding framework and set of principles—complementary to ICAP (and more radical).*

Principle-based use of digital technology to improve STEM learning

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Produced with free software: LuaTeX, ConTeXt, and ImageMagick

University of California, Davis, 18 November 2014
First principles of instruction (Merrill, 2002)

Integration
Application
Demonstration
Activation

Tasks